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CALCULATIONS FOR A CALIBER .50 GUN HAVING 4000 FT/SEC MUZZLE VELOCITY

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Abstract

Calculations were made to determine the conditions necessary in order to obtain 4000 ft/sec in a cal. .50 gun. It was found that a pressure of 87,700 lb/in^2 and a powder potential of 1,805,000 ft lb/lb would be required to produce this velocity in a 45-inch barrel. Calculations have also been made for other barrel lengths and a pressure of 80,000 lb/in^2. The method of calculation is presented in detail.

Contents

I - Statement of the Problem

This problem was presented in a letter from the Chief of Ordnance (OO 472.54/5709; APG 472.5/288, April 23, 1937) as follows:

"Assume that it is desired to obtain a velocity of 4000 f/s with a 45", Caliber .50 barrel using a 750 grain projectile. Make an estimate of the percentage of nitroglycerine, the weight of charge, and the case capacity required. State the factors in the calculations which are doubtful, and propose experiments to evaluate them."

II - Plan of Investigation

This report is based upon computations made by a method developed at Aberdeen Proving Ground. With this method, it is necessary to have the pressure-travel curve of the gun for which the calculations are being made. In the case of a new gun, a pressure-travel curve obtained experimentally from a gun having approximately the same caliber, maximum pressure, weight of charge, etc., may after a suitable transformation, be assumed to be that of the new gun. Pressures obtained from this curve are corrected for the effects of the motion of the powder gas, the friction of the bullet in the bore, and the friction of the gas in the bore. After the forces acting upon the bullet (and therefore the acceleration) have been determined, the velocity is obtained by integration. A detailed account of the method is given in Appendix C to this report.

The directive requested that calculations be made for a 45-inch barrel. As a result of trial calculations, it was found that a bullet travel of 40 inches was the longest which could be used in this barrel without requiring an excessively thick chamber. No other conditions being imposed, a number of solutions were possible, and assumptions had to be made during the progress of the work in order to direct the investigation along what seemed to be the most fruitful channels.

Conditions were first assumed which, it was expected, would secure the desired muzzle velocity with the lowest possible powder potentials. Both high potentials and high pressures were found necessary. By assuming a powder potential of 1,805,000 ft lb/lb it was found that with a 40-inch travel a pressure in excess of 87,700 lb in^2 would be required to produce 4000 ft/sec. (The numerical results are given in detail in Appendix B.)

As this maximum pressure was much in excess of that now produced in Cal. .50 guns (about 56,000 lb/in^2) it was decided to calculate the effects of longer bullet travel and slightly lower pressure. As the powder potentials required were very high, it was also decided to determine the combinations of bore length, muzzle expansion ratio,* and web thickness which would require the lowest potentials.

The conditions assumed for this survey were: Maximum pressure, 80,000 lb/in^2; bullet travels 44 inches, 48 inches, and 52 inches; muzzle expansion ratios 8, 10, and 14; web thickness, such that the grain was completely consumed after 20, 16, and 12 inches of bullet travel.

Due to the method of calculation employed, it was easier to explore the field in this manner than to assume a potential and compute the dimensions of the gun and powder grain required to produce 4000 ft/sec.

As a final step, the charges determined by this method were checked by Roggla's Diagrams. Favorable agreement was secured, considering that two vastly different methods of calculation were used.

III - Principal Numerical Data

Under the assumptions made in the previous section, the conditions under which the required muzzle velocity could be obtained using an available powder potential were determined. By "available potential" is meant a potential that can be secured by a nitrocellulose-nitroglycerine mixture such as the present double base powders. From calculations made at Aberdeen Proving Ground, the potential of powders having different percentages of nitroglycerine was obtained as follows:

Percent Nitroglycerine in Double Base Powder	Potential (ft lb/lb)
20	1,400,000
40	1,680,000
60	1,800,000

* The expansion ratio (η) for any point is defined as the ratio of the sum of the volume swept out by the base of the bullet up to that point and the original free volume, to the original free volume. The muzzle expansion ratio (η_e) is the value when the base of the projectile is leaving the gun.

These values are plotted in Figure 1. Based upon these three points, an extrapolation was performed which indicated that the potential reaches approximately a maximum value of 1,805,000 ft lb/lb in the neighborhood of 65% nitroglycerine. This value was accepted as the maximum available potential for this type of propellant.

It is to be noted that these values of powder potentials are roughly 250,000 ft lb/lb lower than the values given by Crow and Grimshaw* for comparable powders. According to their work, the maximum available potential would be about 2,050,000 ft lb/lb.

The method used in this investigation is a comparison method. That is, the calculations are entirely dependent upon experimental firing and so it is important to know the conditions of that firing, as they show how much the calculated gun differs from the experimental gun.

The comparison firing for the present work was performed with the following values:

Powder	Hercules 1770.5
Potential	1,357,000 ft lb/lb
Charge	240 grains
Web	.0213 in
Initial free volume	.398 in^3
Maximum Pressure	
Piezo-electric	64,000 lb/in^2
Copper	50,000 lb/in^2

Using a bullet travel of 40 inches, a muzzle expansion ratio of 10, and assuming that the web burned through when the bullet had travelled 16 inches, the following potentials were required to obtain 4000 ft/sec at the pressures indicated. (see also Table I):

Maximum Pressure	Potential
85,000 lb/in^2	1,960,000 ft lb/lb
90,000	1,740,000
95,000	1,630,000

* "On the Equation of State of Propellant Gases". A.D. Crow and W.E. Grimshaw, Phil. Trans. Roy. Soc., London, Ser. A, Vol. 230, 1931.

By interpolating it was determined that a maximum pressure of 87700 lb/in^2 would be required in order to use a powder having a potential of 1,805,000 ft lb/lb.

As would be expected, when only 80,000 lb/in^2 is employed, higher potentials and/or longer barrels are required in order to secure the same muzzle velocity. Under the assumptions that the web burned through after 20 inches bullet travel and that the maximum pressure was 80,000 lb/in^2 the potentials required to produce 4000 ft/sec were:

Muzzle Expansion ratio (η_e)	Bullet Travel		
	44"	48"	52"
8	1,850,000	1,810,000	1,725,000
10	1,955,000	1,915,000	1,700,000
14	3,220,000	2,360,000	2,070,000

Complete figures concerning these charges are given in Table II, and a plot of the potential surface is shown in Figure 2.

When the web was assumed to burn through at 16 inches bullet travel, using a maximum pressure of 80,000 lb/in^2 the potentials required to produce 4000 ft/sec were:

Muzzle Expansion ratio (η_e)	Bullet Travel		
	44"	48"	52"
8	2,200,000	1,880,000	1,845,000
10	2,020,000	1,820,000	1,520,000
14	2,890,000	2,400,000	1,995,000

The figures concerning these charges are given in Table III, and the potential surface is plotted in Figure 3.

When it was assumed that the web burned through after 12 inches of bullet travel, still higher potentials were required, and no investigation was made of powders having thinner webs. (See Figure 4 and Table IV).

IV - Discussion

A potential of 1,805,000 ft lb/lb has been adopted as the maximum which may be obtained in double base powders. This value was determined by extrapolation from the known potentials of three powders having different nitroglycerine contents. The extrapolated figure, while it is as accurate as the data permit, is probably not exact. Considerable doubt exists as to whether the potential attains a true maximum. It seems more likely that the potential will continue to increase slightly up to 100% nitroglycerine. Further, the accepted value cannot be considered exact because of the effects of different amounts of stabilizer, volatiles, etc.

Viewed in this light, 1,805,000 ft lb/lb must be considered as approximately the maximum available potential. This value should be sufficiently accurate, as the use of high nitroglycerine content is attended with many practical difficulties, and it is unlikely that such a powder will actually be used.

Even with such a high powder potential, an extremely high pressure is required. This is due to the high acceleration required in order that the bullet shall attain a velocity of 4000 ft/sec at the muzzle, and to the large gas drag caused by the high velocity. If the length of the barrel is increased, less acceleration and a lower pressure will be necessary. A pressure of 87,700 lb/in^2 is required for a bullet travel of 40 inches, while if the pressure is not permitted to exceed 80,000 lb/in^2, a bullet travel of about 45 inches becomes necessary. As the present maximum pressure for Cal. .50 ammunition is about 56,000 lb/in^2, it is apparent that 4000 ft/sec cannot be obtained in a weapon having even approximately the same dimensions as the present 45-inch Cal. .50 barrel.

While it is true that a longer barrel will permit use of lower pressure and lower potential, the barrel cannot be lengthened infinitely. The gas drag increases with the length of the gun, and, due to the form of the gas drag function, a length is soon reached which it is not practicable to exceed. While little numerical work has been done on this subject, the optimum bore length seems to be about 60 inches for the 750 grain, Cal. .50 bullet with a 4000 ft/sec muzzle velocity.

The maximum pressure necessary to secure a given amount of work may be varied by using various muzzle expansion ratios and powder grains of various web thicknesses; thus varying the form of the pressure-travel curve. In Figure 4 each point on the curves represents a pressure-travel curve. These have each produced the same amount of useful work. The effect of varying the form of the pressure curve is seen by the various powder potentials required for any constant bullet travel and constant maximum pressure.

The method used for these calculations does not (as may be seen by examination of Appendix C) take into consideration the increase of energy delivered to the projectile by reason of the enlargement of the chamber over the bore. One calculation was made to determine the magnitude of this effect, and it was found to increase the muzzle velocity by about 3% (in this case).

In using this method of calculation, as with other comparison methods, it is essential to accuracy that only small changes be involved. In the present case, the new gun differs greatly from the one which gave the pressure-travel curve. Greater accuracy can be obtained from the calculations if a pressure record is made under conditions more like those in the 4000 ft/sec gun.

It is found possible to make and desirable to use a powder with a potential of 1,805,000 ft lb/lb (65% NG), then 4000 ft/sec may be obtained if 420 grains of this powder, .0231 inch web, are burned in a cartridge case of 2.436 in^3 capacity and the bullet travels 48 inches in the gun. A piezo-electric pressure slightly over 80,000 lb/in^2 should be expected. As powders having 60% NG have been made which have a potential of 1,800,000 ft lb/lb it should not be impossible to produce a powder of 1,805,000 ft lb/lb potential. Should it be found impracticable to use a pressure of 80,000 lb/in^2 in the preliminary experiments, an equal weight of powder of lower potential could be used and would cause reduced pressures as follows:

Maximum Pressure	Potential	%NG (Approx)
80,000	1,805,000	65
75,000	1,714,000	44
70,000	1,622,000	36
65,000	1,529,000	29

The muzzle velocity obtained when using these lower potentials will, of course, be less than 4000 ft/sec.

V - Resumé

1. It is estimated that a velocity of 4000 ft/sec may be imparted to a 750 grain, Cal. .50 bullet in a 45-inch barrel if a powder potential of 1,805,000 ft lb/lb and a maximum pressure of 87,700 lb/in^2 are used.

2. If a maximum pressure of only 80,000 lb/in^2 is permitted, it is expected that a barrel at least 56 inches long will be required, using the same powder, in order to obtain 4000 ft/sec.

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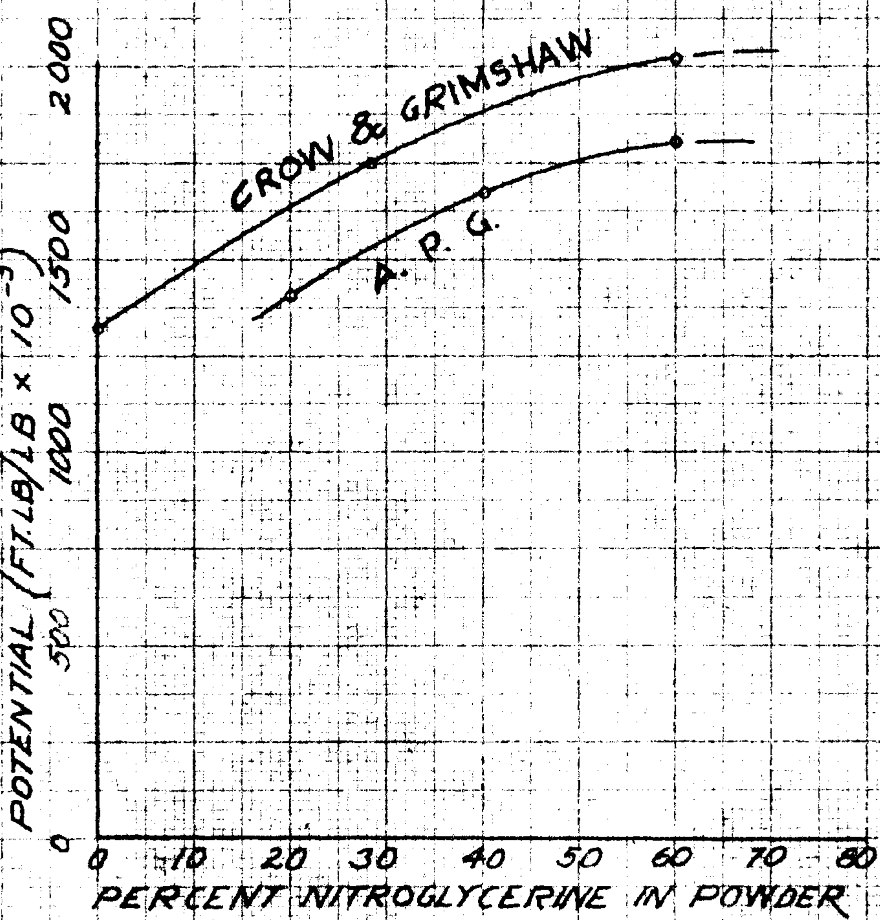
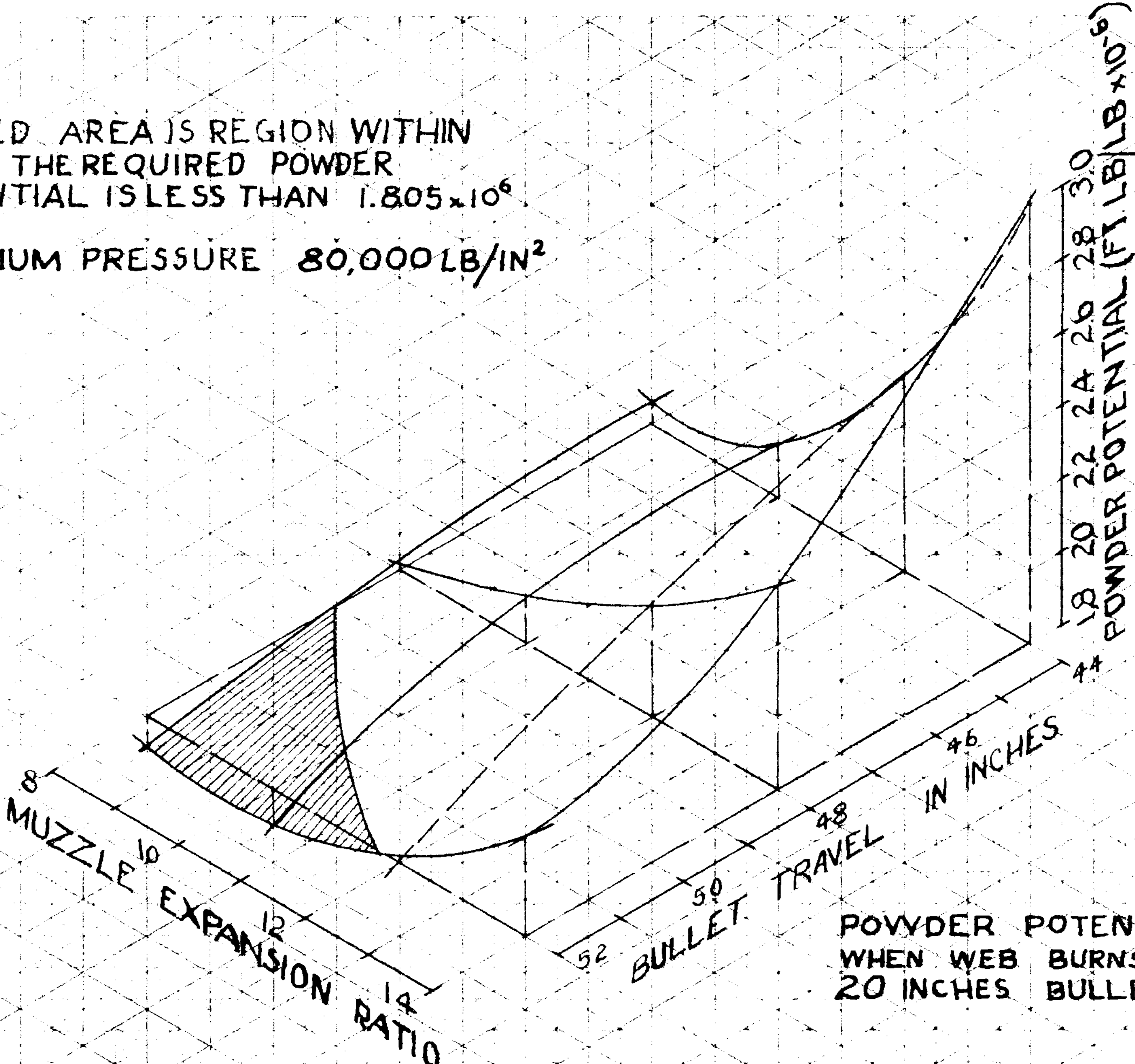


POTENTIALS OF DOUBLE BASE POWDERS

FIG. 1

SHADED AREA IS REGION WITHIN WHICH THE REQUIRED POWDER POTENTIAL IS LESS THAN 1.805×10^6

MAXIMUM PRESSURE $80,000\ LB/IN^2$



POWDER POTENTIAL REQUIRED WHEN WEB BURNS THRU AT 20 INCHES BULLET TRAVEL

FIG. 2

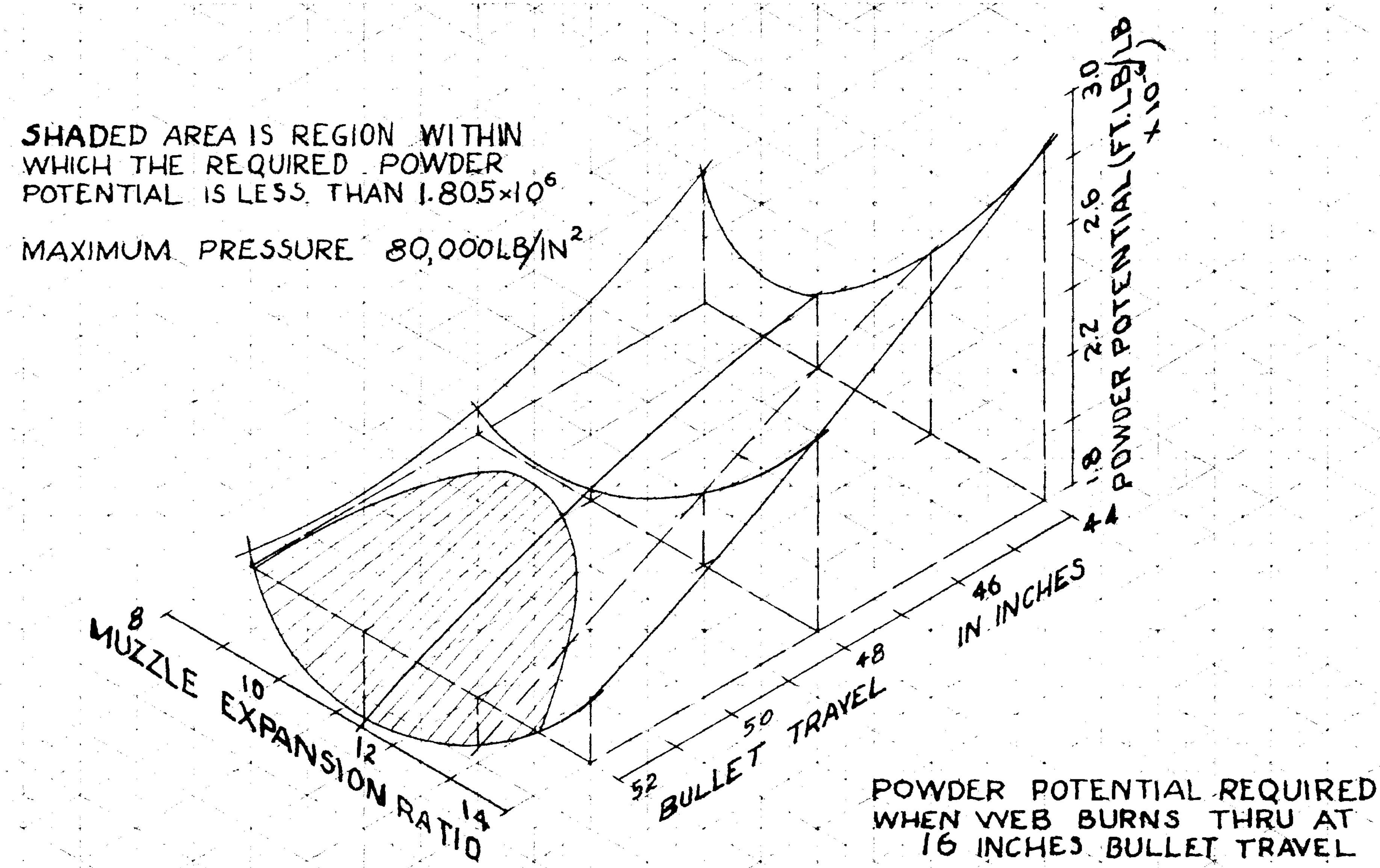
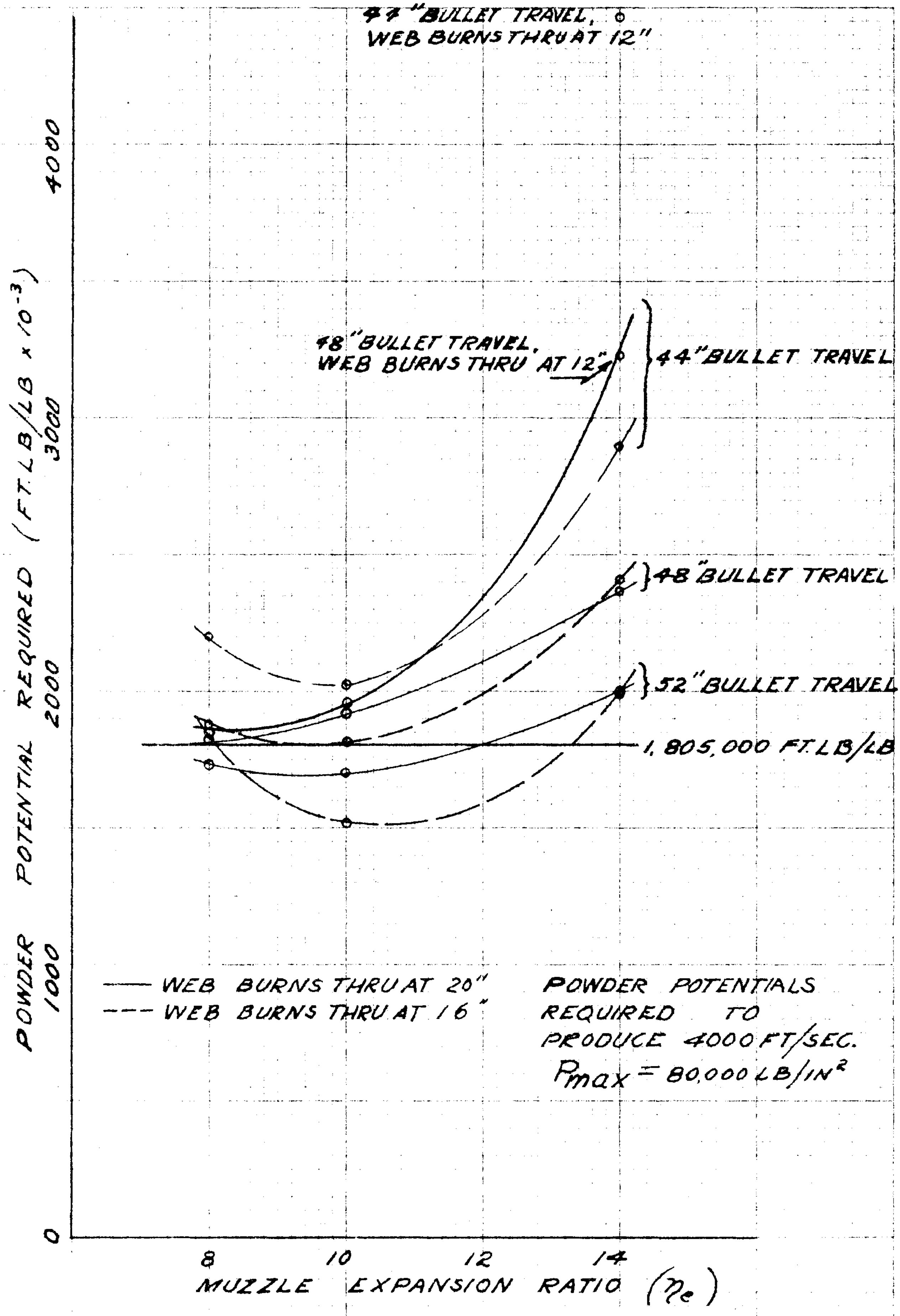

SHADED AREA IS REGION WITHIN WHICH THE REQUIRED POWDER POTENTIAL IS LESS THAN 1.805×10^6
MAXIMUM PRESSURE 80,000LB/IN^2
POWDER POTENTIAL (FT.LB/LB ×10^-6)
1.8
2.2
2.6
3.0
44
46
48
50
52
BULLET TRAVEL IN INCHES
8
10
12
14
MUZZLE EXPANSION RATIO
POWDER POTENTIAL REQUIRED WHEN WEB BURNS THRU AT 16 INCHES BULLET TRAVEL

FIG. 3



44" BULLET TRAVEL, WEB BURNS THRU AT 12"
48" BULLET TRAVEL, WEB BURNS THRU AT 12"
44" BULLET TRAVEL
48" BULLET TRAVEL
52" BULLET TRAVEL
1,805,000 FT.LB/LB
POWDER POTENTIAL REQUIRED (FT.LB/LB × 10^-3)
0
1000
2000
3000
4000
8
10
12
14
MUZZLE EXPANSION RATIO (η_e)
WEB BURNS THRU AT 20"
WEB BURNS THRU AT 16"
POWDER POTENTIALS REQUIRED TO PRODUCE 4000 FT/SEC.
P_{max} = 80,000 LB/IN²

FIG. 4

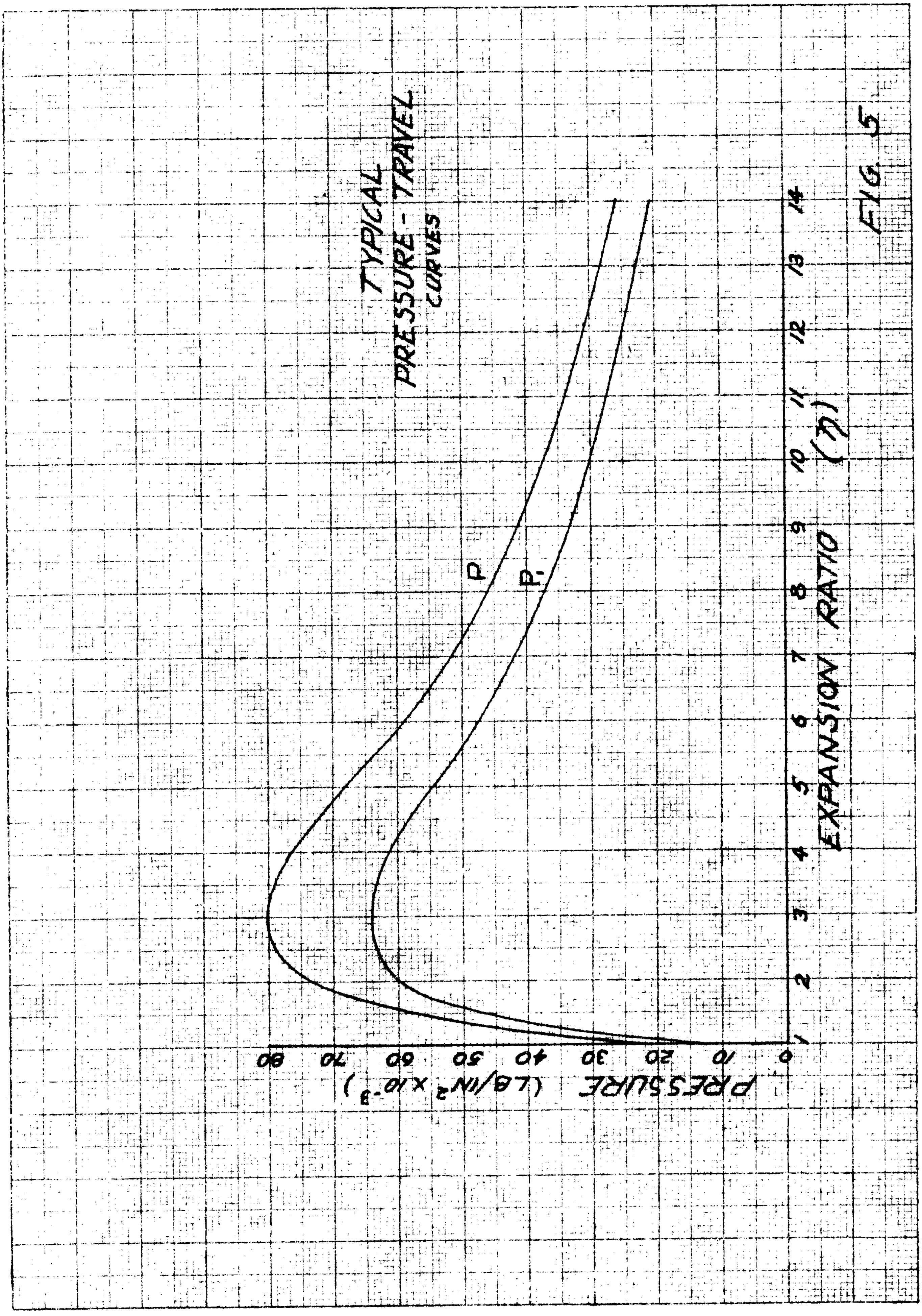

TYPICAL
PRESSURE - TRAVEL
CURVES
P
P1
PRESSURE (LB/IN² x 10⁻³)
0
10
20
30
40
50
60
70
80
EXPANSION RATIO (η)
1
2
3
4
5
6
7
8
9
10
11
12
13
14

FIG 5

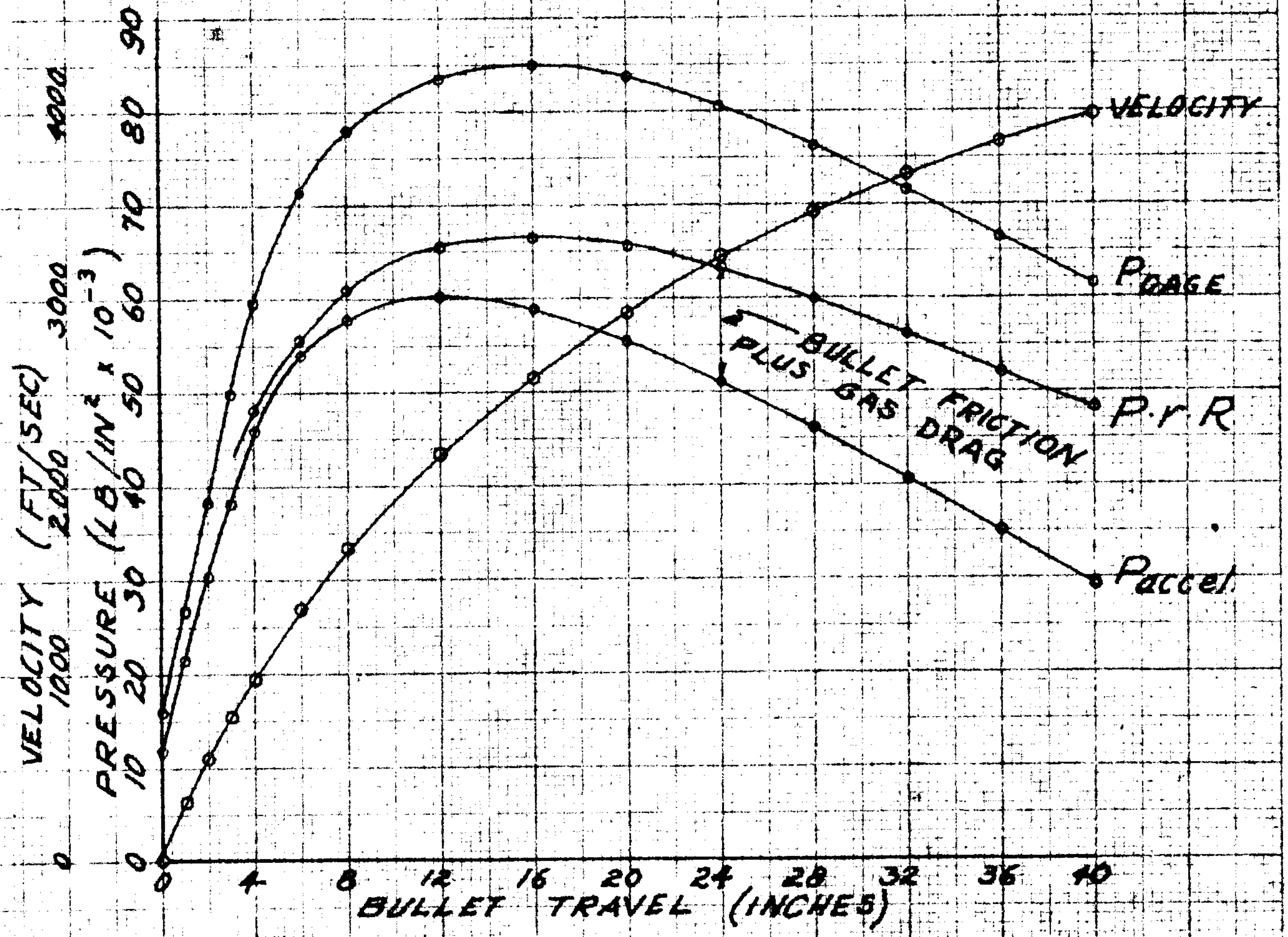

VELOCITY (FT/SEC)
0
1000
2000
3000
4000
PRESSURE (LB/IN² x 10⁻³)
0
10
20
30
40
50
60
70
80
90
BULLET TRAVEL (INCHES)
0
4
8
12
16
20
24
28
32
36
40
VELOCITY
P DRAG
P·r·R
P accel
BULLET FRICTION PLUS GAS DRAG

FIG. 6

APPENDIX B

TABULATED DATA

Note to the Tables

In order to avoid repetition and to simplify the headings in the tables, the units used and general notes are given here.

η_e - The free space expansion ratio at the muzzle.
η_v - The free space expansion ratio at the base of the bullet when the web burns thru.
Charge - Weight, in grains.
Web - Thickness, in inches (single perforated grain).
Potential - Potential of the propellant in ft lb/lb times 10^{-3}.
Case volume - Original free volume plus volume of charge, in cubic inches.
Case length - For travels other than 40 inches the diameter of the case was taken the same as the present cal. .50 case, namely .63 inches, and the length was computed, in inches.
Case diameter - In the case of 40 inches travel, the case was assumed to occupy the remainder of the 45-inch barrel, and the requisite diameter was calculated, in inches.
P_{max} - Maximum piezo-electric pressure lb/in^2.

TABLE I

Bullet travel 40 inches
Case length 5 inches
Muzzle expansion ratio (η_e) = 10
Web burns thru when s = 16 inches (η_v=4.60)

P_{max}	85,000*	90,000	95,000
Charge	290	360	420
Web	.0200	.0213	.0225
Potential (multiply by 10^3)	1,960	1,740	1,630
Case volume	1.623	1.798	1.949
Case diameter	.64	.68	.70

* Routine calculations made by the method of Appendix C gave 4010 ft/sec. When recalculated, using the loading given in this table, and correcting for the effect due to the increased diameter of the chamber, a velocity of 4120 ft/sec was obtained. This is an increase of approximately 3%.

TABLE II

Web burns thru when s = 20 inches

$P_{max} = 80{,}000 \text{ lb/in}^2$

Bullet travel	44"	48"	52"
(a) Muzzle expansion ratio (η_e) = 8			
η_v	4.19	3.91	3.69
Charge	390	420	450
Web	.0228	.0231	.0228
Potential (multiply by 10^3)	1,350	1,810	1,725
Case volume	2.245	2.436	2.626
Case length	7.21	7.82	8.44
(b) Muzzle expansion ratio (η_e) = 10			
η_v	5.08	4.75	4.46
Charge	320	360	410
Web	.0221	.0235	.0230
Potential (multiply by 10^3)	1,955	1,915	1,700
Case volume	1.788	1.978	2.193
Case length	5.74	6.34	7.02
(c) Muzzle expansion ratio (η_e) = 14			
n_v	6.91	6.41	6.00
Charge	160	230	270
Web	.0212	.0216	.0217
Potential (multiply by 10^3)	3,220	2,360	2,070
Case volume	1.084	1.321	1.484
Case length	3.49	4.24	4.76

TABLE III

Web burns thru when s = 16 inches

$P_{max} = 80,000\ lb/in^2$

Bullet travel	44"	48"	52"
(a) Muzzle expansion ratio (η_e) = 8			
η_v	3.55	3.33	3.16
Charge	320	350	370
Web	.0201	.0197	.0197
Potential (multiply by 10^3)	2,200	1,880	1,846
Case volume	2.070	2.260	2.425
Case length	6.64	7.25	7.78
(b) Muzzle expansion ratio (η_e) = 10			
η_v	4.27	4.00	3.77
Charge	280	320	340
Web	.0198	.0197	.0169
Potential (multiply by 10^3)	2,020	1,820	1,520
Case volume	1.688	1.878	2.017
Case length	5.41	6.04	6.47
(c) Muzzle expansion ratio (η_e) = 14			
η_v	5.72	5.33	5.00
Charge	160	200	250
Web	.0190	.0191	.0192
Potential (multiply by 10^3)	2,890	2,400	1,995
Case volume	1.084	1.246	1.432
Case length	3.48	4.00	4.59

TABLE IV

Muzzle Expansion Ratio (η_e) = 14

Web burns thru when s = 12 inches

P_{max} = 80,000 lb/in^2

Bullet travel	44"	48"
η_v	4.55	4.25
Charge	90	130
Web	.0163	.0164
Potential (multiply by 10^3)	4,470	3,220
Case volume	.909	1.071
Case length	2.92	3.44

APPENDIX C

ANALYSIS OF THE METHOD OF CALCULATION

This method is based upon use of a representative pressure-travel curve. If, as in the case where a new gun is being developed, no pressure records are available, it is possible to utilize a record obtained by firing another gun. It is necessary that the new gun and the comparison gun burn the same type of powder and have approximately the same caliber, initial free volume, weight of charge and weight of projectile. If these dimensions are known for both the new and the comparison gun, and a pressure record is available for the comparison gun, then the pressure in the new gun may be calculated by means of Röggla's Equation.* When designing a new gun, however, it is more likely that the maximum pressure will be specified and it will be necessary to calculate the dimensions of the gun and the powder grain. In this case, Röggla's Equation alone is not sufficient to determine all the variables, and must be supplemented by additional methods.

In this case, however, Röggla's Equation makes possible the transformation of the experimental pressure-travel curve into the pressure curve of the new gun.

The pressure when the projectile reaches any point in the bore is a function of the expansion ratio at that point, or, expressed in symbols, $P = f(\eta)$. But, Röggla's Equation is independent of the expansion ratio and

$$\frac{P = f(\eta)}{P_1 = f_1(\eta)} = \text{a constant;}$$

where P and P_1 are the pressures developed by two sets of conditions. Thus it is seen that the experimentally determined pressure-travel curve may be transformed into the pressure-travel curve for the new gun by simple proportion.

This proportionality is shown by Figure 5. The curve having a maximum pressure of [illegible]4,000 lb/in^2 was obtained

* For a complete discussion of Röggla's Equation and its properties see "Neue Diagramme für die Angewandte Innere Ballistik - Pilsen. 1[illegible]. von Edmund Röggla", or A.P.G. Ballistic Laboratory Report No. 48, "Röggla's Equation and its Application to Interior Ballistic Problems." by R. H. Kent.

from a piezo-electric gage record. The other curve was derived from it, and is the curve used in this work when a maximum pressure of 80,000 lb/in^2 was assumed.

This pressure function holds only up to the point where the web burns thru. After the web is burned (or the powder is completely consumed, as single perforated grains are assumed) it is necessary to assume the law of expansion of the gas and to compute the pressures existing at various stages of the expansion. For this purpose the following relation is used

$$PV^{\gamma} = P_1 V_1^{\gamma} \qquad (1)$$

where P and V are respectively the gage pressure and the volume behind the bullet at the instant the web is consumed, and P_1 and V_1 are their values at any subsequent instant. γ is the ratio of the specific heats of the gas, taken as 1.2 for powder gas.

The pressure-travel curve referred to in the preceding paragraphs is assumed to be the reading of the piezo-electric gage. It is necessary to reduce these gage values to the pressure accelerating the bullet. This is done by the relation

$$P_a = (PKr) - (B + \int_0^s D\, ds) \qquad (2)$$

where

P_a = Pressure accelerating the projectile (lb/in^2)

P = Pressure shown by gage (lb/in^2)

K = Correction factor for motion of the powder gas (see below)

r = Correction factor for location of the pressure gage (see below)

B = Bullet friction - Assumed in this case as equivalent to 4800 lb/in^2

$\int_0^s D\, ds$ = Gas drag (see below)

s = Travel of the bullet

The correction factor for the motion of the powder gas, R, corrects the pressure at the breech face to that existing at the base of the projectile.* It is represented by

$$R = \frac{1}{1 + \frac{\epsilon}{2}} \qquad (3)$$

where ϵ = ratio of weight of charge to weight of projectile.

The correction factor for the location of the pressure gage, r, is necessary in order to correct the pressure at the gage to the pressure existing at the breech face. It is obtained from

$$r = 1 + \frac{AK}{M} \quad ^{**} \qquad (4)$$

where

A = Area of bore at the projectile (in^2).

M = Weight of the projectile (grains).

$$K = \frac{CvDA}{aV^2} \qquad (5)$$

C = Weight of the charge (grains).

v = Volume of case between gage and breech (in^3).

D = Distance from breech to gage (in).

a = Diameter of case at gage (in).

V = Volume behind projectile (in^3).

For purposes of these calculations the cartridge case is assumed to be cylindrical and Eqs. (4) and (5) reduce to

$$r = 1 + \frac{CD^2A^2}{MV^2} \qquad (6)$$

* A.P.G. Ballistic Laboratory Report No. 36 - "On the Motion of the Powder Gas." - by R. H. Kent.

** A.P.G. Ballistic Laboratory Report No. 3 - "Analysis of Pressure-Time Curves for the Gerlich Rifle" - Addendum "Derivation of the Travel-Time Curve for Gerlich Rifle from the observed Pressure-Time Curves" by N. F. Ramsey, Jr.

Multiplying the gage pressure, P, by r corrects the gage pressure to the pressure existing at the breech face, and subsequent multiplication by R corrects this value to the pressure which would act at the base of the projectile were it not for the gas drag.

The pressure PrR is opposed by the friction of the bullet in the bore, B, and the gas drag, $\int_0^s D\,ds$. The pressure actually accelerating the projectile, then, is:

$$P_a = PrR - (B + \int_0^s D\,ds),$$ which

will be recognized as Eq. (2). The relations described above are expressed graphically in Figure 6.

The bullet friction, B, was determined by analysis based upon recent firings at various velocities, conducted expressly for the determination of this constant.*

For the caliber .50 bullet, B has the value 4800 lb/in^2.

The Gas Drag, $\int_0^s D\,ds$, is calculated by means of:

$$\int_0^s D\,ds = K\rho V^2(s+\ell) \tag{7}$$

where

D = the specific resistance of each element of the gas column to movement along the bore. D is in such units that the integral is in lb/in^2.

s = the travel of the projectile (in).

K = a constant, in this case taken as .0033. (This constant was determined in conjunction with B above).

ρ = The charge density at any instant (lbs. of powder divided by total cubic inches between projectile and breech).

V = the velocity of the projectile at the point s (ft/sec).

ℓ = an equivalent length of bore having the same surface as the interior of the cartridge case (in).

* For a discussion of the analysis, see A.P.G. Ballistic Laboratory Report No. 3 "Analysis of Pressure-Time Curves for the Gerlich Rifle" by J. R. Lane.

Before performing any numerical work, it is necessary to decide upon the length of bullet travel, the muzzle expansion ratio and the point where the web of the powder is to burn through.

Having the above elements, the typical pressure-travel curve, and an estimated weight of charge, the above equations (2) to (7) enable the calculation of the accelerating pressure, P_a, for several points along the travel, and the plotting of a P_a vs s curve. The muzzle velocity is determined from

$$V^2 = \frac{Ag}{2M} \int_0^s P_a \, ds \qquad (8)$$

where

V = velocity of projectile (ft/sec).

A = area of bore (in^2).

M = weight of projectile (lb).

g = acceleration due to gravity (ft/sec^2).

s = travel of the projectile (ft).

The quantity $\int P_a \, ds$ is determined by mechanical integration of the curve.

If the calculated muzzle velocity agrees with that assumed, it is still necessary to compare the velocities at various points along the bore with those used in computing the gas drag. If necessary, the gas drag is recomputed and a new P_a vs s curve drawn and integrated. If the calculated muzzle velocity does not agree with that assumed, it is necessary to assume a new weight of charge and to recalculate entirely.

After the proper charge is thus determined, the ratio of gage pressure to velocity at each point is plotted against bullet travel and integrated.

The area beneath this curve determines the web thickness required (single perforated grains being assumed). The web thickness burned thru when the bullet has reached any point s is given by

$$r = .000158 \int^{s} \frac{P}{V}\, ds^{*} \qquad (9)$$

where

r = web thickness burned (in).

P = P.E. gage pressure (lb/in^2).

V = Velocity (ft/sec).

s = bullet travel (ft).

When the web burns thru, r becomes half the web thickness required in the grain.

To determine the powder potential that satisfies the conditions imposed by the results of these calculations, Högela's Equation is written in the form:

$$\left[\frac{e_1 C_1 W}{e\ C\ W_1}\right]^2 = \left[\frac{P_1}{P}\right]^{3-2r} \left[\frac{v_{o_1}}{v_o}\right] \left[\frac{C'}{C'_1}\right] \left[\frac{q_1}{q}\right]^2 \qquad (10)$$

* This equation, which is due to Mr. Lane, is derived from the familiar rate-of-burning equation as follows (Terms containing the constant have been neglected):

$$\frac{dr}{dt} = KP + C$$

$$\frac{dr}{dx} \cdot \frac{dx}{dt} + KP + C$$

$$dr = K \frac{P}{V}\, dx + \frac{C}{V}\, dx$$

$$r = K \int \frac{P}{V}\, dx + C \int \frac{dx}{V}\ .$$

where

P = gas pressure.

C = weight of charge.

w = web thickness.

v_o = initial free volume.

G' = corrected weight of the projectile

($G' = M(1 + .014 + \frac{L}{3M})$ where L is the weight of charge and M the weight of the projectile)

q = cross sectional area of the bore.

r = exponent in the rate-of-burning equation.

e = specific energy of the powder.

The terms with subscripts indicate the new conditions, those without, the comparison conditions.

This equation is to be solved for e_1. The value of C_1 used is that which satisfies Eq (8), and of w_1 that determined by Eq (9).